A Simple Model for Instantaneous Radiative Forcing by Optically-Thin Gases

Paulina Czarnecki[1], Robert Pincus[2], and Lorenzo M Polvani[1,2]

1. Columbia University of New York, Department of Applied Physics and Applied Mathematics, New York, United States;
2. Columbia University, Lamont-Doherty Earth Observatory, Palisades, NY, United States

COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science
Lamont-Doherty Earth Observatory
COLUMBIA UNIVERSITY | EARTH INSTITUTE

Introduction

- Simple analytical models for radiative forcing by CO_2 (e.g., [1],[2]) have lent insight into phenomena such as negative forcing by CO_2 at the poles.
- These models rest on the **cooling-to-space-theory**, where all emission of infrared radiation is assumed to originate at one pressure level in the atmosphere where the gas is optically-thick ($\tau\sim 1$).
- Here, we extend this theory to optically-thin gases ($\tau \ll 1$); we **focus on CFC-12**.

Monochromatic Forcing

We start with the monochromatic Schwartzschild's equation:

(pre-industrial; present)

$$\mathscr{F}(\nu) = F^{net\downarrow}_{present} - F^{net\downarrow}_{PI} = OLR_1(\nu) - OLR_2(\nu)$$

(Planck function; surface emission; atmospheric emission)

$$= \left[B(T_s)\mathcal{T}_{1,s} - \int_{p_{TOA}}^{p_s} B(p)\frac{d\mathcal{T}_1}{dp}dp\right] - \left[B(T_s)\mathcal{T}_{2,s} - \int_{p_{TOA}}^{p_s} B(p)\frac{d\mathcal{T}_2}{dp}dp\right]$$

(Transmissivity $\mathcal{T} = e^{-\tau}$; $\mathcal{T}_1 - \mathcal{T}_2$)

$$= B(T_s)\Delta\mathcal{T}_s - \int_{p_{TOA}}^{p_s} B(p)\frac{d}{dp}\Delta\mathcal{T}(p)dp.$$

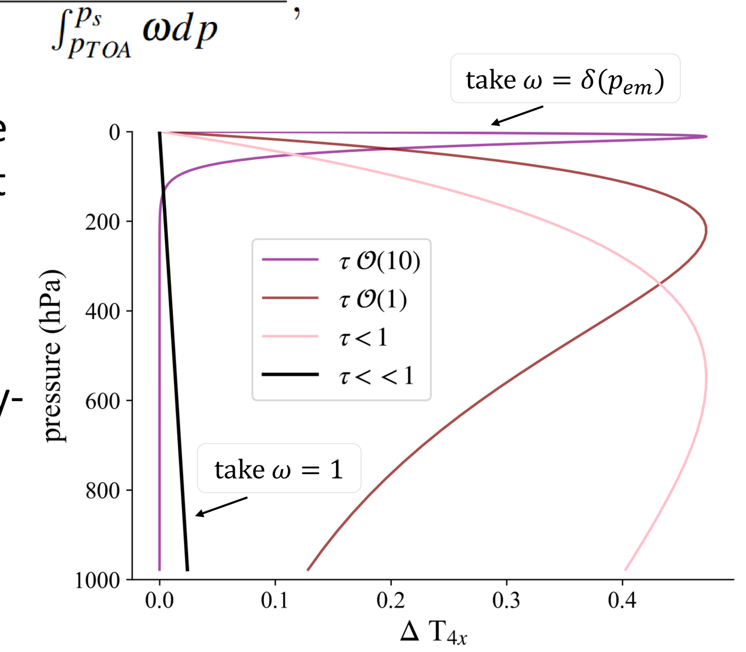
Assuming the gas emits from some average atmospheric temperature, we pull the Planck function out of the integral:

$$\mathscr{F}(\nu) = B(T_s)\Delta\mathcal{T}_s - \overline{B}\int_{p_{TOA}}^{p_s} \frac{d}{dp}\Delta\mathcal{T}(p)dp$$
$$= B(T_s)\Delta\mathcal{T}_s - \overline{B}\Delta\mathcal{T}_s.$$

We let

$$\overline{B} = \frac{\int_{p_{TOA}}^{p_s}\omega(p)B(p)dp}{\int_{p_{TOA}}^{p_s}\omega dp},$$

and when $\tau \ll 1$ we take $\omega = 1$. This is equivalent to asserting that **transmissivity changes linearly in pressure.** Note that for an optically-thick gas such as CO_2, taking $\omega = \delta(p_{em})$ returns the cooling-to-space model [e.g., 1-3].



Finally, as $\tau \ll 1$, we approximate transmissivity with the Taylor series expansion around 0: $e^{-\tau} \approx 1 - \tau \Rightarrow \Delta\mathcal{T} \approx \Delta\tau$.

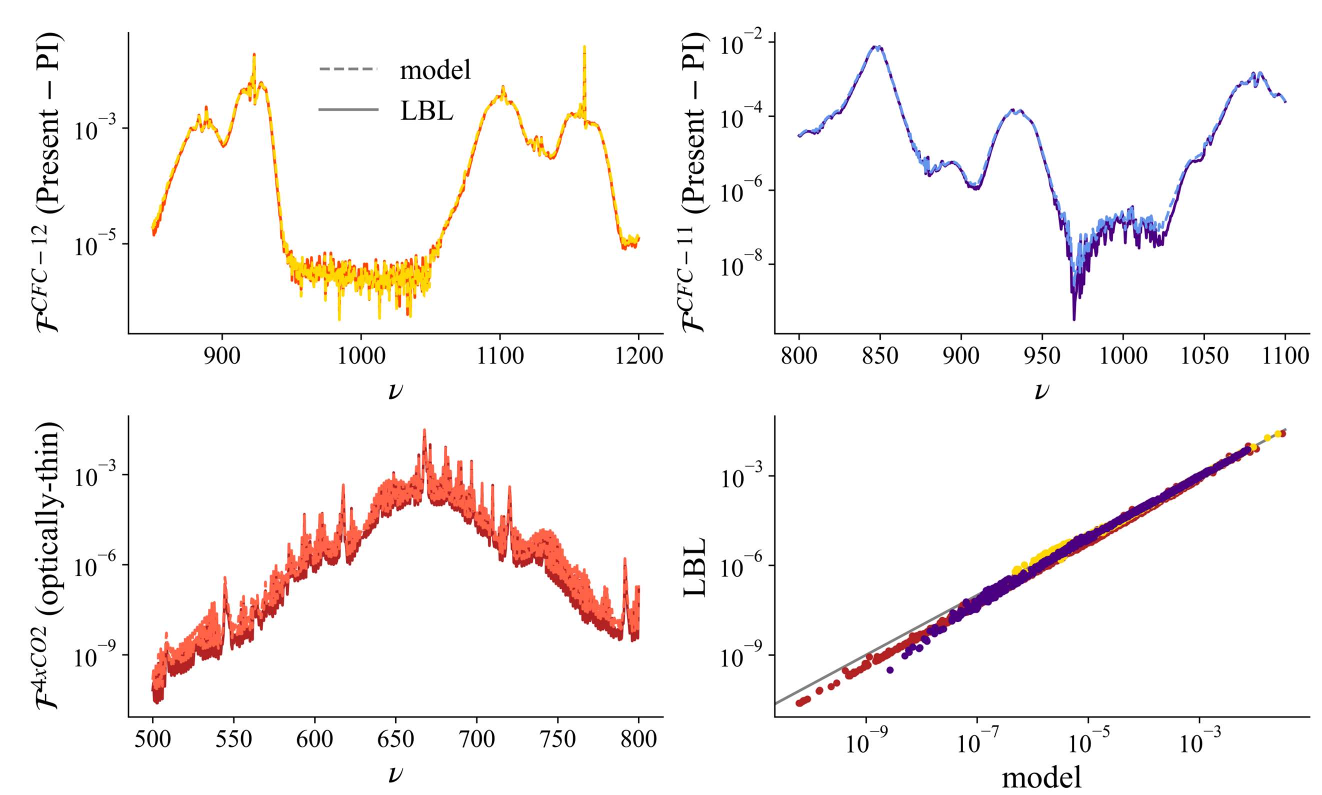
Then the model becomes

(surface emission; change in total optical thickness)

$$\mathscr{F}(\nu) \approx \left(B(T_s) - \overline{B}\right)\Delta\tau_s.$$

(mean Planck function in pressure)

Monochromatic Forcing Model Validation



The model is agnostic to absorption coefficient shape and predicts forcing **monochromatically** across a wide range of gases, including optically-thin CO_2, with small errors.

Spectral Integration

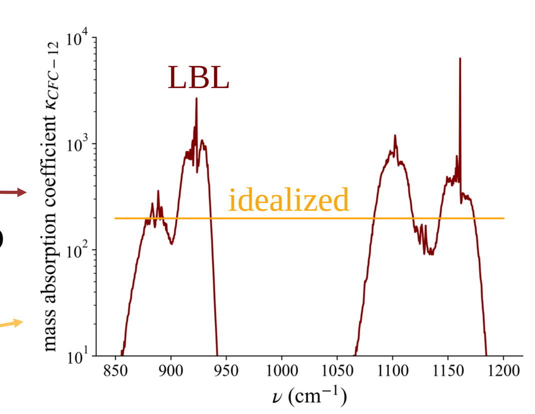
As we take the forcing to be linear in optical depth τ, and idealize τ as linear in pressure for reference absorption coefficient κ:

(2-stream coefficient; gas concentration)

$$\tau(p) = \int_{p_{TOA}}^{p_s} D\kappa_{ref}(\nu)\frac{q}{g}dp \approx D\kappa_{ref}(\nu)\frac{q}{g}p,$$

we can appeal to the Mean Value Theorem to parameterize

$$\kappa^*_{ref} = \frac{\int_{\nu_1}^{\nu_2}\kappa_{ref}d\nu}{\nu_2 - \nu_1}.$$



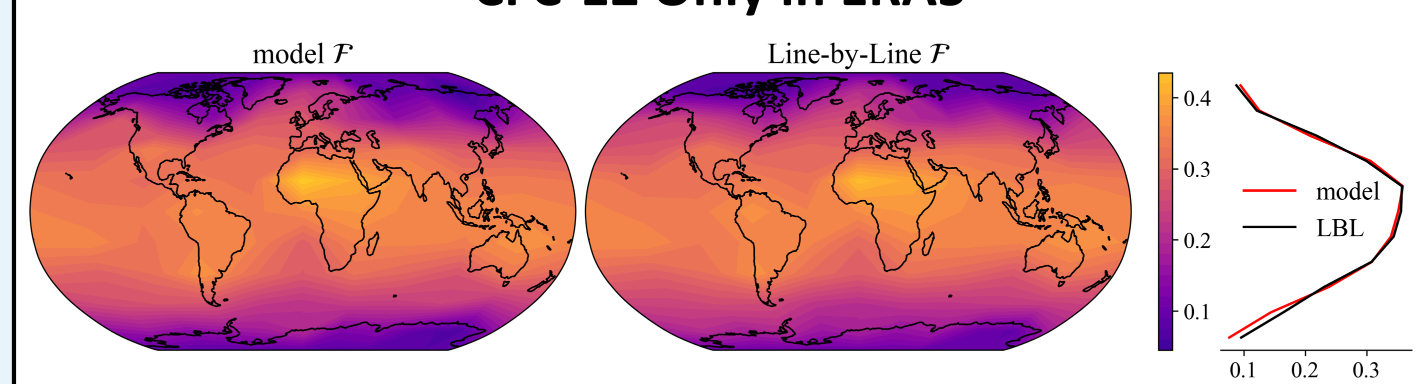
Then the band-integrated forcing can be modeled

(surface emission at central wavenumber; band-mean atmospheric thickening)

$$\mathscr{F} = \left(B(T_s,\nu^*) - \overline{B}(\nu^*)\right)\Delta\tau^*_s(\delta\nu).$$

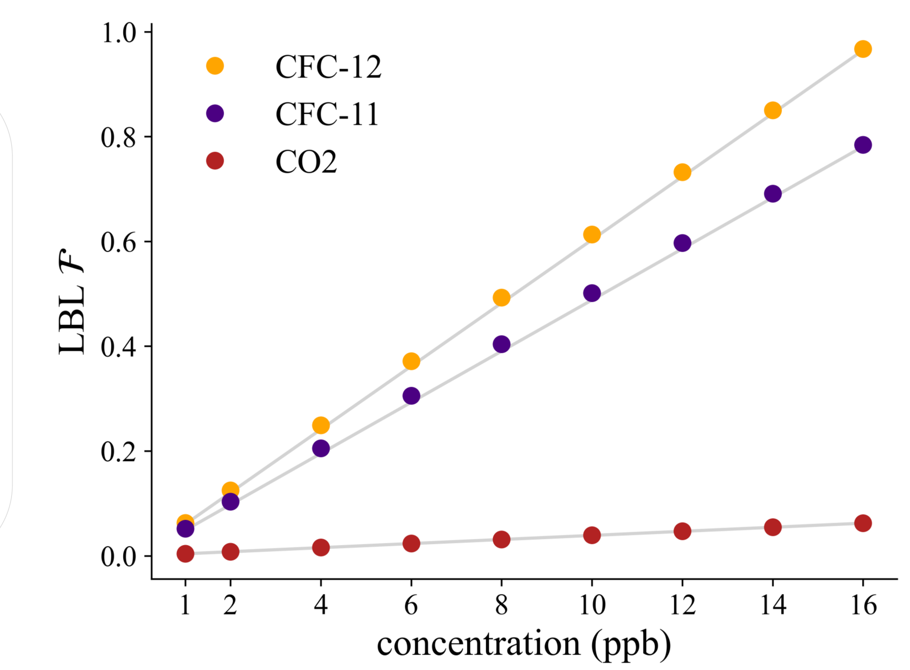
(mean atmospheric emission at central wavenumber; band width)

CFC-12 Only in ERA5



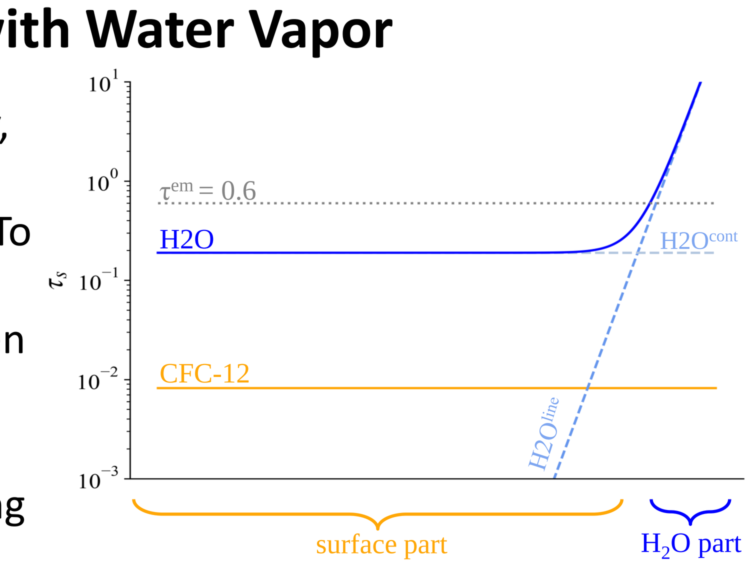
Dependence of Forcing on Concentration

Monochromatic and band-integrated forcing are both linear in optical depth, and thus **linear in concentration** for well-mixed, optically-thin gases.



Overlap with Water Vapor

Following [1], monochromatically, we assume that the level where $\tau_{H_2O} = 0.6$ replaces the surface. To integrate spectrally, we:



- assume **no correlation** between the absorption coefficients of CFC-12 and H_2O.
- Sort $\kappa^{line}_{H_2O}$ and $\kappa^{continuum}_{H_2O}$ along the spectrum.
- Model their optical depths according to [1,4], and add them together.
- Split the band into two parts: where $\tau_{H_2O} < 0.6$ and we ignore water vapor, and where $\tau_{H_2O} > 0.6$.
- Calculate the mean emission temperature of the H2O [1].

Then our model becomes:

(mean change in CFC-12 surface optical depth)

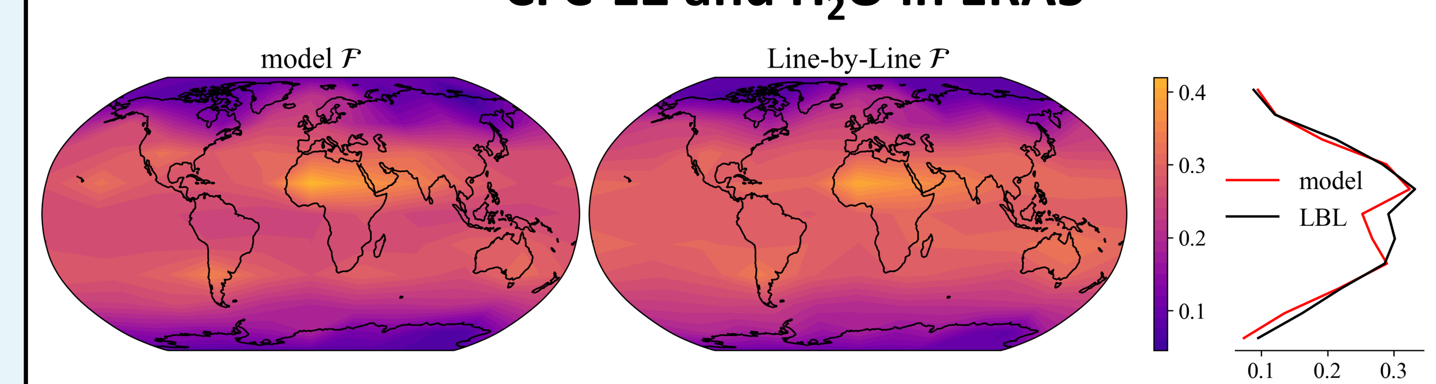
$$\mathscr{F} = \left(B(T_s,\nu^*) - \overline{B}(\nu^*)\right)\Delta\tau^*_s(\delta\nu_s)$$

(band-mean surface Planck term; mean change in CFC-12 optical depth at H_2O emission pressure)

$$+ \left(B(T^{H2O}_{em},\nu^*) - \overline{B}(\nu^*)\right)\Delta\tau^*(p^{H2O}_{em})(\delta\nu_{H2O}).$$

(mean Planck term at H_2O emission temperature)

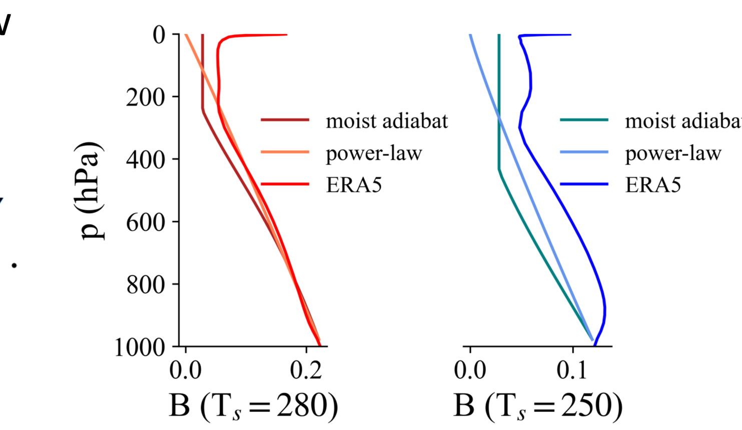
CFC-12 and H_2O in ERA5



Idealized Atmospheres

We create atmospheres with a moist adiabatic lapse rate in the troposphere and an isothermal stratosphere at 200K, following [1]. Then, borrowing from [5], we idealize the Planck function profile as a power law in pressure, where γ describes the lapse rate:

$$B = B(T_s)\left(\frac{p}{p_s}\right)^{\gamma}.$$



Then $\overline{B}$ can be integrated directly:

$$\overline{B} = \frac{1}{p_s}\int_{p_{TOA}}^{p_s} B(T_s)\left(\frac{p}{p_s}\right)^{\gamma}dp = \frac{B(T_s)}{1+\gamma}$$

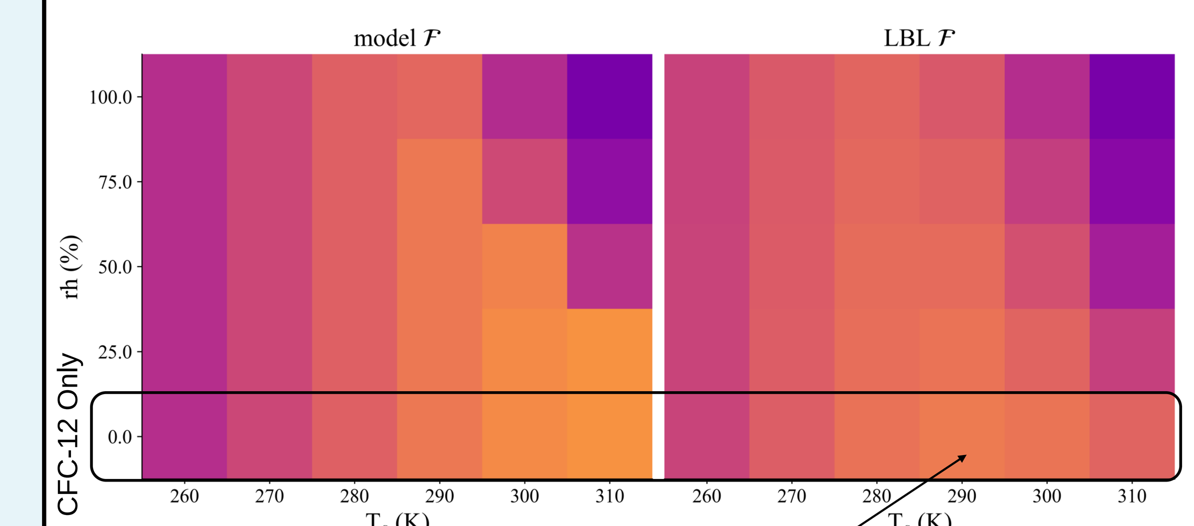
And our model becomes

(lapse rate scaling; mean change in optical depth)

$$\mathscr{F} = \left(\frac{\gamma}{1+\gamma}\right)B(T_s,\nu^*)\Delta\tau^*_s(\delta\nu).$$

(Planck function at the surface)

→ this formulation highlights the dependence of the forcing on the **lapse rate**, or the **vertical structure of the atmosphere.**



In the LBL calculations for idealized atmospheres, CFC-12 forcing peaks at ~290 K, which our model doesn't capture. We also don't see this in ERA5 profiles.

Conclusion

- Our simple model highlights the dependence of optically-thin gas forcing on the **temperature structure** and the **total optical thickening** of the atmosphere.
- The model shows that optically-thin gas forcing is **linear in concentration**, and this dependence holds monochromatically.

References

[1] Jeevanjee et al., (2021). *J. Cim.*
[2] Wilson and Gea-Banacloche, (2012). *Amer. J. Phys.*
[3] Jeevanjee and Fueglistaler, (2020b). *J. Atmos. Sci.*
[4] Jeevanjee and Fueglistaler, (2020a). *J. Atmos. Sci.*
[5] Koll et al., (2023). *J. Atmos. Sci*