Radiative transfer method used in STORM

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The radiation field in the atmosphere is determined by single and multiple scattering at air molecules, aerosols and droplets and by interaction with the surface. Starting point for the investigation of single and multiple scattering effects in an atmosphere-surface system is the radiative transfer equation[1]

$$\begin{aligned}(\mu\tfrac{d}{d\tau}+1)I(\tau,\mu,\phi) = \;& \pi\varpi_0(\tau)\widetilde{P}(\mu,\mu_0,\phi,\phi_0)S_0 e^{-\frac{\tau}{\mu_0}} \\ & +\varpi_0(\tau)\textstyle\int_0^{2\pi}\int_{-1}^{+1}\widetilde{P}(\mu,\phi,\mu',\phi')I(\tau,\mu',\phi')d\mu' d\phi'\end{aligned} \tag{1}$$

with μ=$\cos\theta$ (figure 1). $I(\tau,\mu,\phi)$ is the Stokes vector of the diffuse scattered radiation field at boundary τ. θ is the zenith angle. S_0 is the spectral solar flux incident at the top of atmosphere, μ_0 and ϕ_0 are the cosine of zenith angle and azimuth of incident solar flux, μ und ϕ are the corresponding values in viewing direction. $\varpi_0(\tau)$ is the single scattering albedo of the scatterers in the considered layer. $\widetilde{P}$ is the phase matrix rotated into the incident and viewing planes. This presentation of the radiative transfer equation is valid for plane parallel atmospheres. The local dependence is described only by the optical thickness τ, which is an integral value over height. Refractive characteristics of the atmosphere are not considered. The first term on the right side in equation 1 describes the first or single scattering process of the radiation in the atmosphere. The second term describes multiple scattering and includes all following scattering processes, that contribute to the Stokes vector I into direction (μ,ϕ).

The phase matrix $\widetilde{P}$ is determined for rotationally symmetric particles and the azimuthal dependence can be separated. The radiative transfer equation is expanded into a Fourier

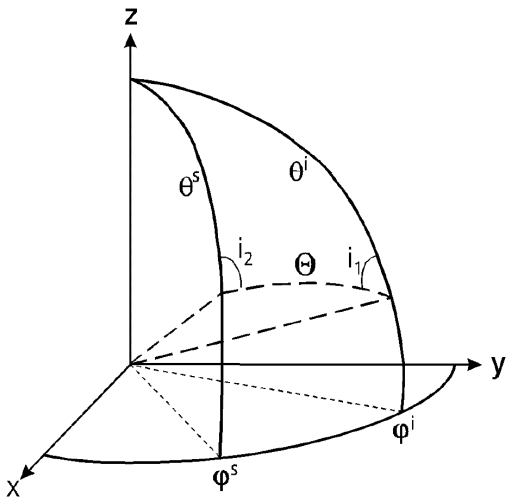

Figure 1. Geometry of radiative transfer equation: θ_i - incident zenith angle; θ_s - viewing zenith angle; ϕ - corresponding azimuth; Θ - scattering angle. The principal plane corresponds to $\phi = 0°$ and $\phi = 180°$.

series, which leads to an equation system of N+1 independent equations, that depend only on the cosine of the incident and viewing zenith angle μ_0 and μ respectively:

$$\begin{aligned}(\mu\tfrac{d}{d\tau}+1)I_l(\tau,\mu) = \;& \varpi_0(\tau)\widetilde{P_l}(\mu,\mu_0)S_0 e^{-\frac{\tau}{\mu_0}} \\ & +\varpi_0(\tau)\pi(1+\delta_{0l})\textstyle\int_{-1}^{+1}\widetilde{P_l}(\mu,\mu')I_l(\tau,\mu')d\mu'\end{aligned} \tag{2}$$

To treat the downwelling (μ>0) and upwelling (μ<0) radiation fields $I^+$and I^-separately the equation 2 is rewritten:

$$\begin{aligned}(\mu\tfrac{d}{d\tau}+1)I_l^+(\tau,\mu) = \;& \varpi_0(\tau)\widetilde{P_l}(\mu,\mu_0)S_0 e^{-\frac{\tau}{\mu_0}} \\ & +\varpi_0(\tau)\pi(1+\delta_{0l})[\textstyle\int_0^1 \tilde{P}_l(\mu,\mu')I_l^+(\tau,\mu')d\mu' \\ & +\textstyle\int_0^1 \tilde{P}_l(\mu,-\mu')I_l^-(\tau,\mu')d\mu']\end{aligned} \tag{3}$$

$$\begin{aligned}(-\mu\tfrac{d}{d\tau}+1)I_l^-(\tau,\mu) = \;& \varpi_0(\tau)\widetilde{P_l}(-\mu,\mu_0)S_0 e^{-\frac{\tau}{\mu_0}} \\ & +\varpi_0(\tau)\pi(1+\delta_{0l})[\textstyle\int_0^1 \tilde{P}_l(-\mu,\mu')I_l^+(\tau,\mu')d\mu' \\ & +\textstyle\int_0^1 \tilde{P}_l(-\mu,-\mu')I_l^-(\tau,\mu')d\mu']\end{aligned} \tag{4}$$

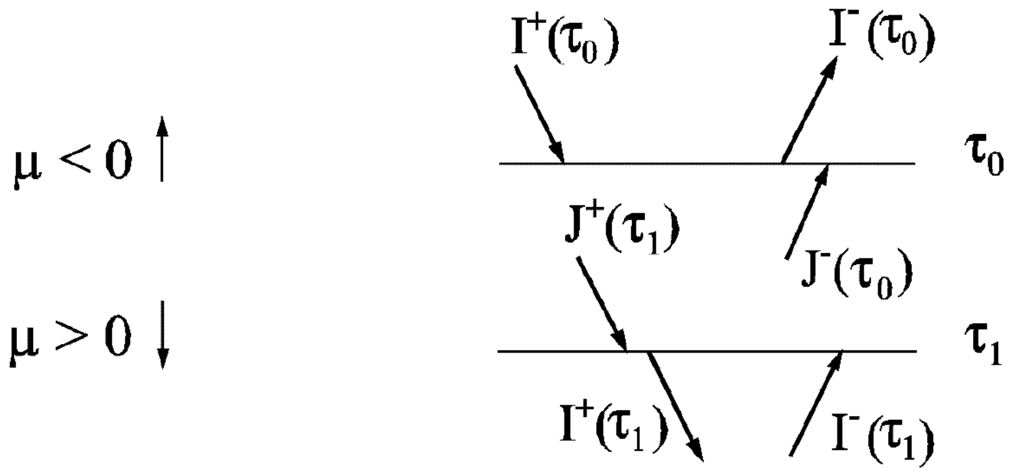

Figure 2. *Schematic presentation of values used in the interaction principle*

with μ, $\mu' > 0$. The integrals in the equations are solved by Gauss-Lobatto quadrature:

$$\int_0^1 f(\mu)d\mu = \sum_{i=1}^{m} f(\mu_i)c_i \tag{5}$$

which allows the computation of the Stokes vector at $\mu = \pm 1$. c_i are the quadrature weights at μ_i.

To solve the radiative transfer equation for a plane parallel inhomogeneous atmosphere the matrix operator method[2] is used. The basis of this method is the interaction principle applied to the transmission, reflection and source operators for a single layer. A plane parallel vertically inhomogeneous atmosphere-surface system can be approximated by a number of homogeneous layers. The boundaries of these homogeneous layers are labelled from the top downward by their optical thicknesses τ_0, τ_1, τ_2, $\ldots$. The downward radiation $I^+(\tau_0, \theta)$ is incident from above on the layer (τ_0, τ_1). The upward radiation $I^-(\tau_1, \theta)$ is incident from below on this layer (fig. 2). Furthermore, $I^+(\tau_1, \theta)$ and $I^-(\tau_0, \theta)$ are emerging downward and upward from the layer, respectively. $J^+(\tau_1, \theta)$ and $J^-(\tau_0, \theta)$ are the downward and upward sources inside the layer. The relations between the upward and downward directed incident and emerging radiation and the sources of the considered layers are described by the interaction principle and the diffuse transmission operators t and diffuse reflection operators r.

$$
\begin{aligned}
I_1^+ &= t_{01} I_0^+ + r_{10} I_1^- + J_{01}^+ \\
I_0^- &= r_{01} I_0^+ + t_{10} I_1^- + J_{10}^-
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
I_2^+ &= t_{12} I_1^+ + r_{21} I_2^- + J_{12}^+ \\
I_1^- &= r_{12} I_1^+ + t_{21} I_2^- + J_{21}^-
\end{aligned}
\tag{7}
$$

$t_{ij} = t(\tau_i, \tau_j)$ describes the transmission from boundary τ_i to boundary τ_j and $r_{ij} = r(\tau_i, \tau_j)$ describes the reflection at boundary τ_j in direction τ_i. Applying the interaction principle for the combined layer (τ_0, τ_2)

$$
\begin{aligned}
I_2^+ &= t_{02} I_0^+ + r_{20} I_2^- + J_{02}^+ \\
I_0^- &= r_{02} I_0^+ + t_{20} I_2^- + J_{20}^-
\end{aligned}
\tag{8}
$$

the transmission, reflection and source operators t_{02}, t_{20}, r_{02}, r_{20}, J_{02}^+ and J_{20}^- can be derived from the operators of the single layers using simple mathematical manipulations

$$
\begin{aligned}
t_{02} &= t_{12}(E - r_{10} r_{12})^{-1} t_{01} \\
r_{20} &= r_{21} + t_{12}(E - r_{10} r_{12})^{-1} r_{10} t_{21} \\
J_{02}^+ &= J_{12}^+ + t_{12}(E - r_{10} r_{12})^{-1} (J_{01}^+ + r_{10} J_{21}^-) \\
t_{20} &= t_{10}(E - r_{12} r_{10})^{-1} t_{21} \\
r_{02} &= r_{01} + t_{10}(E - r_{12} r_{10})^{-1} r_{12} t_{01} \\
J_{20}^- &= J_{10}^- + t_{10}(E - r_{12} r_{10})^{-1} (J_{21}^- + r_{21} J_{01}^-)
\end{aligned}
\tag{9}
$$

with $E = [\delta_{ij}]$. Application of this procedure to all layers of the atmopshere gives the transmission, reflection and source operators for the whole atmosphere expressed through the corresponding operators of the single layers. The operators of single layers are retrieved by a coefficient comparison of the radiative transfer equation written in matrix form with the interaction principle for infinitesimal layers and are estimated by

$$\begin{aligned}
t_{10} &= E - \Gamma_l^{++}(\tau)\Delta_{10} + o(\Delta_{10}) \\
t_{01} &= E - \Gamma_l^{--}(\tau)\Delta_{10} + o(\Delta_{10}) \\
r_{01} &= \Gamma_l^{+-}(\tau)\Delta_{10} + o(\Delta_{10}) \\
r_{10} &= \Gamma_l^{-+}(\tau)\Delta_{10} + o(\Delta_{10}) \\
\Sigma_{01}^{+} &= \Sigma_l^{+}(\tau)\Delta_{10} + o(\Delta_{10}) \\
\Sigma_{01}^{-} &= \Sigma_l^{-}(\tau)\Delta_{10} + o(\Delta_{10})
\end{aligned} \tag{10}$$

with $\tau_0 \leq \tau \leq \tau_1$ and $\Delta_{10} = \tau_1 - \tau_0$. $o(\Delta_{10})$ is a rest term with $\lim_{\tau_1 \to \tau_0} \frac{o(\Delta_{10})}{\Delta_{10}} \to 0$ and with

$$\begin{aligned}
\Gamma_l^{++}(\tau) &= M^{-1}[E - \varpi_0(\tau)\pi(1+\delta_{0l})\widetilde{P_l}^{++}(\tau)c] \\
\Gamma_l^{+-}(\tau) &= M^{-1}\varpi_0(\tau)\pi(1+\delta_{0l})\widetilde{P_l}^{+-}(\tau)c \\
\Gamma_l^{-+}(\tau) &= M^{-1}\varpi_0(\tau)\pi(1+\delta_{0l})\widetilde{P_l}^{-+}(\tau)c \\
\Gamma_l^{--}(\tau) &= M^{-1}\left[E - \varpi_0(\tau)\pi(1+\delta_{0l})\widetilde{P_l}^{--}(\tau)c\right] \\
\Sigma_l^{+}(\tau) &= M^{-1}J_l^{+}(\tau) \\
\Sigma_l^{-}(\tau) &= M^{-1}J_l^{-}(\tau)
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
M &= [\delta_{ij}\mu_i] \\
c &= [\delta_{ij}c_i] \\
J_l^{+}(\tau) &= [J_l^{+}(\tau,\mu_i,\mu_j)] = \varpi_0(\tau)S_0[\widetilde{P}_l^{++}(\tau,\mu_i,\mu_j)e^{-\frac{\tau}{\mu_j}}] \\
J_l^{-}(\tau) &= [J_l^{-}(\tau,\mu_i,\mu_j)] = \varpi_0(\tau)S_0[\widetilde{P}_l^{-+}(\tau,\mu_i,\mu_j)e^{-\frac{\tau}{\mu_j}}].
\end{aligned} \tag{12}$$

With these equations the reflection, transmission and source operators can be calculated for the whole atmosphere-surface system and the upward and downward diffuse radiation field can be derived at every desired boundary in the atmosphere.

The surface is treated here as an additional layer with special reflection operators and no transmission. For an isotropically reflecting surface the model of a Lambertian reflector is applied, which transforms any incident radiation distribution into a uniform distribution. The reflection at a wind rough water surface is described by Fresnel's law combined with a surface wind speed dependent probability distribution of the slope of the water surface developed by Cox and Munk[3].

Based on the Matrix-Operator Method the STOkes vector Radiative transfer Model (STORM) was developed in a joint project of DLR and Institut für Weltraumwissenschaften of the Freie Universität Berlin[4]. STORM calculates the full Stokes vector in a plane parallel, multi layered atmosphere. The scattering characteristics of aerosols are determined by Mie theory. The surface represents a Lambertian reflector or a wind ruffled water surface. The result of one calculation is the upward and downward directed Stokes vector (I, Q, U, V) for one wavelength at a desired number of sun incident and viewing angles at different heights in the principal plane and other azimuth angles.

References

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