Zeeman splitting - implementation in radiative transfer

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- Theoretical overview of the Zeeman effect
- RTE and the Zeeman effect
- Conclusions

Zeeman effect

$$H = H_0 + H_{LS} - \vec{\mu_J} \cdot \vec{B}$$

- H - total Hamiltonian.
- H_0 - unperturbed Hamiltonian.
- H_{LS} - spin-orbit coupling Hamiltonian.
- $\vec{\mu_J}$ - total magnetic moment.
- $\vec{B}$ - external magnetic field.

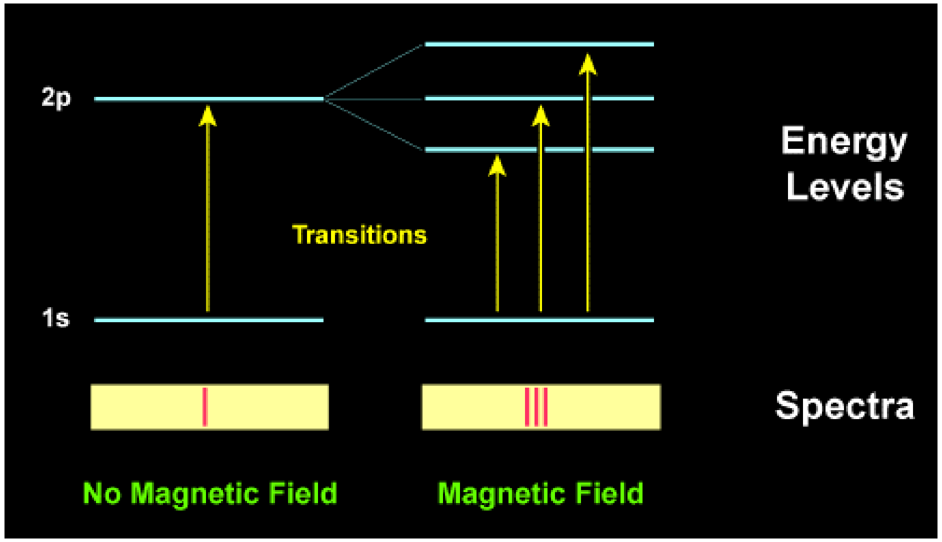
$$H\Psi = (E + E_B)\Psi = (E + < -\vec{\mu_J} \cdot \vec{B} >)\Psi = (E + \mu_B \; B \; g_J \; M_J)\Psi$$

- E - unperturbed by $\vec{B}$ energy level .
- E_B - energy level perturbation due to $\vec{B}$.
- μ_B - $Bohr$ magneton, $\mu_B = \frac{\hbar e}{2 m_e c}$.
- B - magnitude of the external magnetic field.
- g_J - $Land\acute{e}$ *g* factor, $g_J = 1 + \frac{(J+1)J + (S+1)S - (L+1)L}{2J(J+1)}$.
- M_J - magnetic quantum number, $M_J = -J, ..0, ...J$.

Zeeman effect splitting of the energy levels

Energy levels spacing

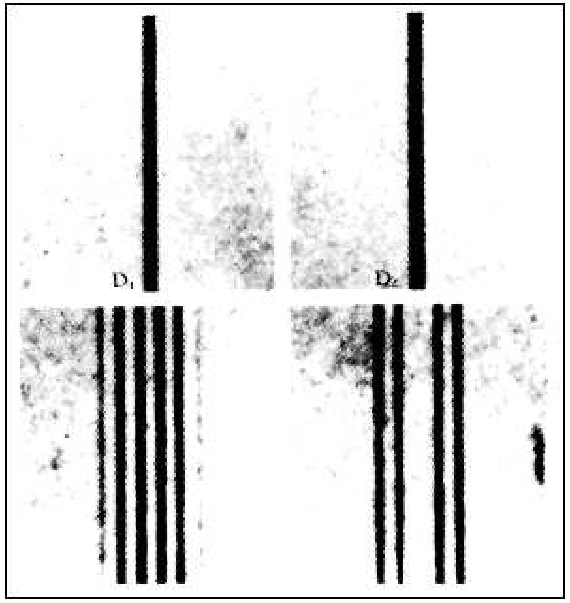
$$\Delta E = \Delta E_B = \mu_B \, B \, g_J$$



Zeeman effect splitting of the spectral lines

Zeeman's original photo of the effect:

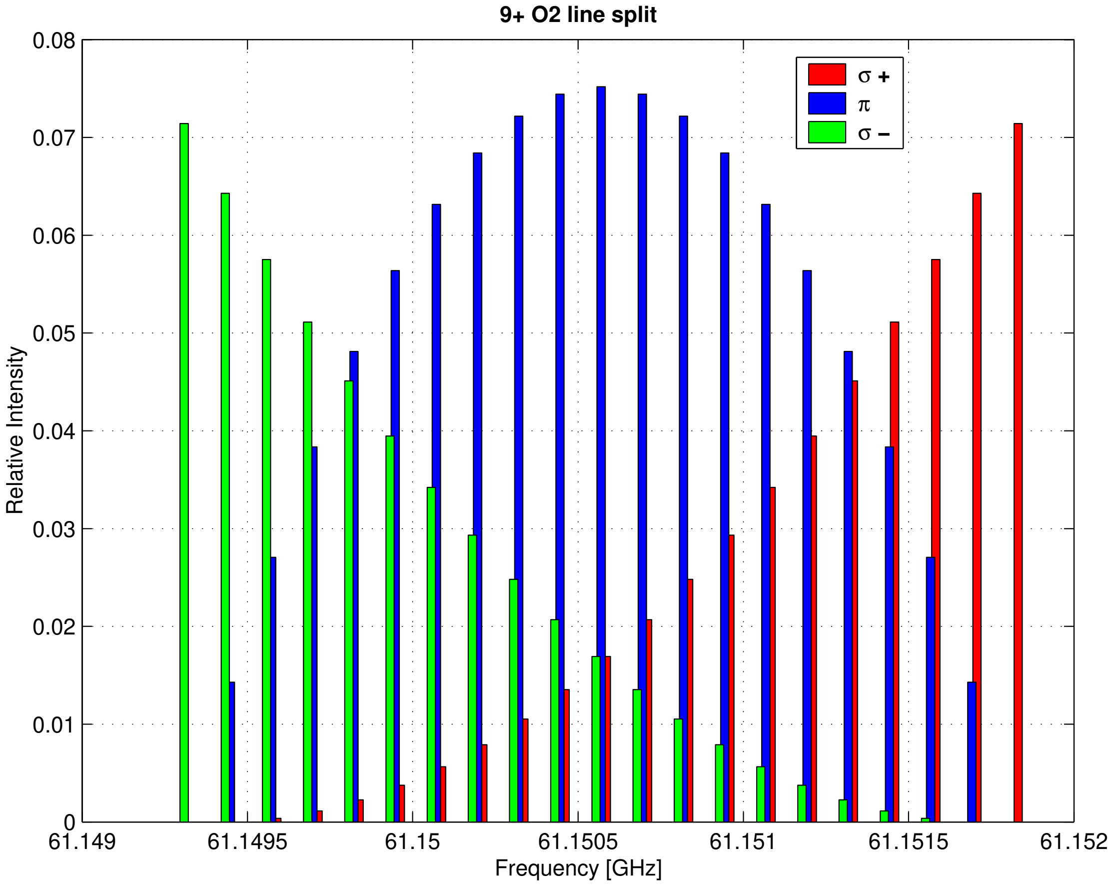
- unsplit lines above (no magnetic field)
- split lines below by the magnetic field



Zeeman effect splitting of the spectral lines

Selection rules: ΔJ = 0, ± 1

ΔM_J = 0 (π polarization), ± 1 ($\sigma \pm$ polarization)



Maxwell equations in the frequency domain

Source-free and electrically isotropic medium

$$\begin{aligned}
\nabla \times \vec{E}(\vec{r},\omega) &= -\, i\omega\ \vec{B}(\vec{r},\omega) = -\, i\omega\ \hat{\mu}(\omega) \cdot \vec{H}(\vec{r},\omega) \\
\nabla \times \vec{H}(\vec{r},\omega) &= \ i\omega\ \vec{D}(\vec{r},\omega) = \ i\omega\ \epsilon_0 \vec{E}(\vec{r},\omega) \\
\nabla \cdot \vec{D}(\vec{r},\omega) &= 0 \\
\nabla \cdot \vec{B}(\vec{r},\omega) &= 0
\end{aligned}$$

- $\vec{E}(\vec{r},\omega)$, $\vec{D}(\vec{r},\omega)$ - *electric field strength and displacement*
- $\vec{H}(\vec{r},\omega)$, $\vec{B}(\vec{r},\omega)$ - *magnetic field strength and flux density*
- $\hat{\mu}(\omega)$, ϵ_0 - *magnetic permeability (tensor) and electric permittivity*

Anisotropy and wave propagation

Magnetic susceptibility tensor $\hat{\chi}(\omega)$ *:*

$$\hat{\mu}(\omega) = \mu_0(\hat{I} + \hat{\chi}(\omega))$$

Assumptions:

- *plane wave*
- *slight magnetic anisotropy of the medium* $|\chi_{ik}(\omega)| << 1$

$\Longrightarrow$ *2D wave propagation equation (*$H_s \approx 0,\ s - propagation\ \ path$*):*

$$\frac{\partial^2}{\partial s^2}\vec{H}(s,\omega) - k_0^2[\hat{I} + \hat{\chi}(\omega)] \cdot \vec{H}(s,\omega) = 0$$

Complex propagation tensor

$$G(\hat{\omega})^2 = -k_0^2[\hat{I} + \hat{\chi}(\omega)] \quad \Longrightarrow \quad \hat{G}(\omega) \approx ik_0[\hat{I} + \frac{1}{2}\hat{\chi}(\omega)] = \hat{A}(\omega) + i\hat{B}(\omega)$$

- $\hat{G}(\omega)$ - *propagation tensor*
- $\hat{A}(\omega)$ - *attenuation tensor*
- $\hat{B}(\omega)$ - *phase tensor*

Solution to the wave propagation equation:

$$\begin{pmatrix} H_x \\ H_y \end{pmatrix}(s,\omega) = e^{-\hat{G}(\omega)s} \cdot \begin{pmatrix} H_x \\ H_y \end{pmatrix}(0,\omega)$$

Coherence matrix

$$\hat{S} = < \vec{E}_p \vec{H}^* >$$

where

$$\vec{E}_p(s,\omega) = \begin{pmatrix} -E_y \\ E_x \end{pmatrix} (s,\omega) = -\frac{\hat{G}(\omega)}{i\omega\epsilon_0} \cdot \vec{H}(s,\omega)$$

Maxwell equations for $\vec{E}_p$ *and* $\vec{H}$*:*

$$\begin{aligned} \frac{\partial}{\partial s}\vec{E}_p(s,\omega) &= -\, i\omega\, \hat{\mu}(\omega) \cdot \vec{H}(s,\omega) \\ \frac{\partial}{\partial s}\vec{H}(s,\omega) &= i\omega\epsilon\, \vec{E}_p(s,\omega) \end{aligned}$$

Coherence matrix propagation

$$\frac{\partial}{\partial s}\hat{S}(s,\omega) + \hat{G}(\omega)\cdot\hat{S}(s,\omega) + \hat{S}(s,\omega)\cdot\hat{G}^*(\omega) = 0$$

Using $\hat{S} = \hat{P} + i\hat{Q} \;\Leftrightarrow \hat{P} = \hat{P}^*$, $\hat{Q} = \hat{Q}^*$:

$$\frac{\partial}{\partial s}\hat{P}(s,\omega) + \hat{G}(\omega)\cdot\hat{P}(s,\omega) + \hat{P}(s,\omega)\cdot\hat{G}^*(\omega) = 0$$

where $Tr(\hat{P}) = <(\vec{E}\times\vec{H})_s> = <(\vec{S})_s>$

Brightness temperature matrix propagation

Raleigh-Jeans approximation:

$$\hat{T}_B(\nu,\ T,\ \theta,\ \phi) = \frac{c^2}{2\nu^2\ k}\hat{B}(\nu,\ T,\ \theta,\ \phi)$$

Assumptions:

- *no scattering from the medium*
- *no thermal emission from the medium*

$$\frac{\partial}{\partial s}\hat{T}_B(s,\omega) + \hat{G}(\omega)\cdot\hat{T}_B(s,\omega) + \hat{T}_B(s,\omega)\cdot\hat{G}^*(\omega) = 0$$

General solution

With thermal emission from the medium $\hat{E}_M$:

$$\frac{\partial}{\partial s}\hat{T}_B(s,\omega) + \hat{G}(\omega)\cdot\hat{T}_B(s,\omega) + \hat{T}_B(s,\omega)\cdot\hat{G}^*(\omega) = \hat{E}_M(\omega)$$

Upon negligible scattering from the medium:

$$\hat{E}_M(\omega) = \hat{G}(\omega)\cdot\hat{T}_E(\omega) + \hat{T}_E(\omega)\cdot\hat{G}^*(\omega)$$

where $\hat{T}_E$ is the temperature of thermal equilibrium of the medium .

$$\Longrightarrow\ \hat{T}_B(s,\omega) = e^{-\hat{G}(\omega)s}\cdot\hat{T}_B(0,\omega)\cdot e^{-\hat{G}^*(\omega)s} + \\ + \hat{T}_E(\omega) - e^{-\hat{G}(\omega)s}\cdot\hat{T}_E(\omega)\cdot e^{-\hat{G}^*(\omega)s}$$

Stokes vector and $\hat{\mathrm{T}}_{\mathrm{B}}$

In a circular basis:

$$\begin{aligned} s_0 &= T_B^{circ}(11) + T_B^{circ}(22) \\ s_1 &= 2Re[T_B^{circ}(12)] = 2Re[T_B^{circ}(21)] \\ s_2 &= 2Im[T_B^{circ}(12)] = 2|Im[T_B^{circ}(21)]| \\ s_3 &= T_B^{circ}(11) - T_B^{circ}(22) \end{aligned}$$

where $[s_k]_{k=0}^{3}$ - *Stokes vector components.*

$\hat{\mathrm{G}}$ matrix by Zeeman effect

$$\hat{G}(\omega) = ik_0 \left[\hat{I} + \sum_{\Delta M_J=-1}^{1} \hat{P}_{\Delta M}(\Phi) \sum_{M} N_{M,\Delta M}(\omega) \right]$$

where

$$\begin{aligned} \hat{P}_0(\Phi) &= \frac{\sin^2\Phi}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \\ \hat{P}_\pm(\Phi) &= (1/2) \begin{pmatrix} (1 \pm \cos\Phi)^2 & \sin^2\Phi \\ \sin^2\Phi & (1 \mp \cos\Phi)^2 \end{pmatrix} \end{aligned}$$

where $N_{M,\Delta M}(\omega)$ account for the line shape and intensity of the individual components of the split line, and Φ=$\angle(\vec{B},\vec{e}_z)$.

Propagation equation for the Stokes vector

$$\frac{\partial}{\partial s}\vec{I} = \hat{K} \cdot \vec{I} + T\,\vec{a}$$

where

- $\vec{I}$ *- Stokes vector,* $\vec{I} = (I_0, I_1, I_2, I_3)$
- $\hat{K}$ *- extinction matrix*
- $\vec{a}$ *- absorption vector*
- T *- kinetic temperature of the medium*

Extinction matrix and emission term

$$\hat{K} = -\frac{1}{2}\begin{pmatrix} 2Re[Tr(G)] & 4Re[G_{kl}] & 0 & 2Re[G_{11}-G_{22}] \\ 4Re[G_{kl}] & 2Re[Tr(G)] & -2Im[G_{11}-G_{22}] & 0 \\ 0 & 2Im[G_{11}-G_{22}] & 2Re[Tr(G)] & 4Im[G_{kl}] \\ 2Re[G_{11}-G_{22}] & 0 & -4Im[G_{kl}] & 2Re[Tr(G)] \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} 2Re[Tr(G)] \\ 4Re[G_{kl}] \\ 0 \\ 2Re[G_{11}-G_{22}] \end{pmatrix}$$

where G_{kl} is the non-diagonal element of $\hat{G}$, ($G_{kl} = G_{lk}$).

Conclusions

Current models of Zeeman effect need improvements by:

- *taking into account of scattering*
- *modeling better the global and height variation of Earth's* $\vec{B}$.

References

Lenoir, W. B., Propagation of partially polarized waves in a slightly anisotropic medium, Journal of Applied d Physics, Vol. 38, p. 5283-5290, 1967

Lenoir, W. B., Microwave spectrum of molecular oxygen in the mesosphere. Journal of Geophysical Research, Vol.73, No.1, p. 361-376, 1968

Rosenkrantz, P. W., and Staelin, D. H., Polarized thermal microwave emission from oxygen in the atmosphere, Radio Science, Vol.23, No.5, p. 721-729, 1988